GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: I

Paper –M 101: ALGEBRA – I

Model Paper

Time: 3Hours Max.Marks:60

SECTION-A (Essay Questions)

Answer **ALL** the questions. Each question carries **12** Marks. **4 x 12 = 48 M**

1. **(a)** Let G be a group acting on a set X. Then the set of all orbits in X under G is a partition of X. For any x ∈ X there is a bijection Gx → $\frac{G}{G_x}$ and hence $|G_x|=[G:G_x]$.Therefore if X is a finite set, $|X|=\sum_{x\in C}[G:G_x]$, where C is a subset of X containing exactly one element from each orbit.

(b) Find the number of different necklaces with p beads, p prime where the beads can have any of n different colours.

(**OR)**

2. **(a)** Let G be a group . If G is solvable, then every subgroup of G and every homomorphic image of G are solvable. Conversely, if N is a normal subgroup of G such that N and $\frac{G}{N}$ are solvable, then G is solvable.

(b) State and prove Jordan – Holder theorem.

3. State and prove fundamental theorem of finitely generated abelian group.

(OR)

4. State and prove Sylow 2^{nd} theorem .

5. State and prove one – one correspondence theorem in rings and ideals.

(OR)

6. Let R be a commutative principal ideal domain with identity. Then prove that any nonzero ideal P ≠ R is prime if and only if it is maximal.

7. Prove that Every Euclidean Domain is a UFD.

(OR)

8. Prove that if R is a UFD then R$[x]$ is also UFD.

SECTION-B(Short Answer Questions)

9. Answer any **three** questions. Each question carries **4** marks. **3 x 4=12 M**

(a) Every group of order P^2 (p is prime) is abelian.

(b) State and prove sylow third theorem.

(c) Define Nilpotent ideal and given example.

(d) Define Euclidean Domain, PID, UFD.

(e) State and prove Gauss Lemma.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: I

Paper-M 102: Real Analysis -I

Model Paper

Time:3 Hours **Max.Marks: 60**

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **12** Marks. **4 x 12 = 48 M**

1. a) Show that every neighbourhood is an open set.

   b) Let $\{E_n\}$ be a finite or infinite collection of sets E_α. Then show that

   $(\bigcup_\alpha E_\alpha)^C = \bigcap_\alpha (E_\alpha^C)$.

OR

2. a) Suppose Y⊂ X. Show that a subset E of Y is open relative to Y if and only if E=Y∩G for some open subset G of X.

   b) Show that closed subsets of compact sets are compact

3. a) Suppose $\{S_n\}$ is monotonic.Then show that $\{S_n\}$ converges if and only if it is bounded.

   b) Show that $\sum \frac{1}{n^p}$ converges if p > 1 and diverges if p ≤ 1

OR

4. a) Show that e is irrational.

   b) State and prove Root Test

5. Let f be a Continuous mapping of a compact metric space X into a metric space Y.

Then show that f is uniformly continuous on X.

OR

6. a) Let f be a continuous real function on the interval [a,b] .If f(a)<f(b) and if c is a number such that f(a)<c<f(b),then show that their exist a point x ∈ (a,b) such that f(x)=c.

b) Let f be a monotonically increasing on (a,b). Then show that f(x^+) and f(x-) exist at every point of x of (a,b)

7. State and prove Taylor's theorem.

OR

8. a) Let f be defined on [a, b]; if f has a local maximum at a point x∈ (a,b), and if f'(x) exists then show that f'(x)=0.

b) Suppose f is real differentiable function on [a,b] and suppose f'(a) < λ < f'(b). Then show that there is a point x ∈ (a, b) such that f'(x) = λ.

SECTION-B(Short Answer Questions)

9. Answer any **three** questions. Each question carries **4** marks. **3 x 4=12 M**

a. Let k be a positive integer. If { I_n} is a sequence of k-cells such that $I_n \supset I_{n+1}$, n=1,2,3,……, then show that $\bigcap_{n=1}^{\infty} I_n$ is non empty.

b. If $\{s_n\}$ and $\{t_n\}$ are complex sequences, and $\lim_{n\to\infty} s_n$ = s , $\lim_{n\to\infty} t_n$=t , then show that $\lim_{n\to\infty} s_n\, t_n$ = st .

c. Prove that in any metric space X, every convergent sequence is a Cauchy sequence.

d. If f is a continuous mapping of a compact metric space X into $\mathbb{R}^k$, then show that f(X) is closed and bounded.

e. Let f be defined on [a, b]. If f is differentiable at a point x ∈ [a, b] , then showthat f is continuous at x.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: I

Paper-M 103: DIFFERENTIAL EQUATIONS

Model Paper

Time: 3 Hours Max.Marks: 60 M

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **12** Marks. **4 x 12 = 48 M**

1. If y_1(x) and y_2(x) are the linearly independent solutions of the homogeneous equation y''+ P(x) y' + Q(x) y = 0 on the interval [a,b],then prove that c_1y_1(x) + c_2y_2(x) is a solution of the differential equation. Also prove that the wronskian
   W =W($y_{1,}y_2$) is either identically zero or never zero on [a,b].

OR

2. a) If y_1(x) = x is the solution for the differential equation x^2y''+xy'-y=0,then find the general solution of the differential equation

   b) Find the particular solution for the equation y''+2y'+y=e^{-x}log x

3. State and prove Sturm comparision theorem.

OR

4. Let y(x) & z(x) are non trivial solutions of y''(x) +Q(x)y=0 and Z''(x)+R(x)Z=0 where q(x) & r(x) are the positive function such that Q(x) > r(x) then y(x) vanishes atleast once between any two successive zero's of Z(x)

5. Find the Frobenius series solution and the general solution for the differential equation
   4x^2y'' +8x^2y'+(4x^2+1)y=0

OR

6. Consider the function y=$(1+x)^p$ where p is the arbitrary constant. It is easy to indicated particular solution of the (1+x)y'= py, y(0)=1

7. a) Construct successive approximations for the solution of initial value problem
   y' = x+y, y(0)=1.

b) If W(t) represents wronskian of two non trivial solutions of $x'=a_1(t)x + b_1(t)y$, $y'=a_2(t)x + b_2(t)y$ on [a,b], then show that W(t) is either identically equal to zero or nowhere zero on [a,b].

OR

8. State and prove Picard's theorem and use the picard's method to solve y'=5y, y(0)=1.

SECTION-B(Short Answer Questions)

9. Answer any **three** questions. Each question carries **4** marks. **3 x 4=12 M**

a. Show that y= $c_1x + c_2x^2$ is the general solution of x^2y''- 2xy'+2y=0 on any interval not containing zero and find the particular solution for y(1)=3,$y'(1)$=5
b. Find the particular solution of y''+ y= cosecX
c. If Q(x)<0 and u(x) is a non trivial solution u''+Q(x)u=0 .Then u(x) has atmost one zero.
d. Find the second degree using power series equation
e. (1+x^2) y''+2xy'-2y=0
f. Show that f(x, y)= $y^{\frac{1}{2}}$ does not satisfy lipschitz condition on the rectangle.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: I

Paper-M 104: TOPOLOGY

Model Paper

Time:3 Hours Max.Marks: 60 M

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **12** Marks. **4 x 12 = 48 M**

1. a) Show that every separable metric space is second countable.

   b) Let X be a topological space and A a subset of X. Show that A = A ∪ D(A)

OR

2. a) Show that the set R^n of all n-tuples of real numbers is a real Banach space with respect to coordinate wise addition and scalar multiplication and the norm defined by $\|X\|^2 = \sum_{i=1}^{n} | x_i |^2$.

   b) Define a nowhere dense set in a topological space X. Show that a closed set in X is nowhere dense if and only if its complement is everywhere dense in X.

3. a) State and prove Lebesgue's covering lemma.

   b) Show that a metric space is compact if and only if it is complete and totally bounded.

OR

4. a) Show that every compact metric space has the Bolzano -Wierstrass property .

   b) Show that every continuous mapping of a compact metric space X into a metric space Y is uniformly continuous on X.

5. a) A Topological space is a T_1-Space⇔each point is a closed set.

   b) Prove that every compact subspace of a Hausdroff space is closed.

OR

6. State and prove Urhyson's Lemma.

7. a) Show that a subspace of the real line $\mathbb{R}$ is connected $\Leftrightarrow$ it is an interval.

   b) Show that any Continuous image of a connected space is connected.

OR

8. a) Let X be a topological space and A be a connected subspace of X. If B is a subspace of X such that $A \subseteq B \subseteq \bar{A}$ then prove that B is connected, in particular $\bar{A}$ is connected.

   b) Show that the components of a totally disconnected space are its points.

SECTION-B(Short Answer Questions)

9. Answer any **three** questions. Each question carries **4** marks. **3 x 4=12 M**

a. If T_1 and T_2 are two topologies on a non-empty set X, show that $T_1 \cap T_2$ is also a topology on X.
b. Define a weaker topology and give an example of it.
c. State Heine – Borel theorem.
d. Define T_1-space and Hausdorff space.
e. Define connected space and give one example.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: I

Paper-M 105: DISCRETE MATHEMATICS

Model Paper

Time:3 Hours ____ Max.Marks:60 M

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **12** Marks. **4 x 12 = 48 M**

1. a) I) Define the terms i) Digraph ii) Degree of a graph

II) Prove that the maximum number of edges in a graph with n-vertices is n(n-1)/2.

b) Prove that for any set of positive integers $n_1, n_2, \ldots\ldots n_k$,

$$\sum_{i=1}^{k} n_i^2 \leq \left[\sum_{i=1}^{k} n_i\right]^2 - (k-1)\left[2\sum_{i=1}^{k} n_i - k\right].$$

OR

2. a) Prove that a graph is bipartite if and only if it contains no odd cycles.

b) Prove that there are $\frac{1}{2}$(n+1) pendent vertices in any binary tree with n vertices.

3. a) Define an Eulerian graph and prove that a graph G is Eulerian if and only if every vertex of G is of even degree.

b) Let G be a connected graph with n vertices, where n > 2.Let u and v be a pair of distinct non adjacent vertices of G such that d(u) + d(v) ≥ n. Then prove that G + uv is Hamiltonian if and only if G is Hamiltonian.

OR

4. a) Prove that a graph is Hamiltonian if and only if its closure is Hamiltonian.

b) Explain the Kruskal algorithm and Prisms algorithm.

5. a) Let (L,Λ,V) be an algebraic lattice .If we define X ≤ Y ⇔ X Λ Y = X (or X ≤ Y ⇔ X V Y = Y), then prove that (L, ≤) is a lattice ordered set.

b) Prove that a lattice L is distributive if and only if it does not contain a sub lattice isomorphic to the pentagon lattice.

OR

6. a) Prove that a lattice L is distributive if and only if for all x, y ,z ∈ L ,

$(x \wedge y) \vee (y \wedge z) \vee (z \wedge x) = (x \vee y) \wedge (y \vee z) \wedge (z \vee x)$.

b) Prove that if L is a distributive lattice, then each x∈ L has atmost one complement.

7. a) State and prove Demorgan's laws in a Boolean algebra.

b) State and prove Representation theorem for finite Boolean algebras.

OR

8. a) Let B be a Boolean algebra and let I be a non-empty subset of B. then the following conditions are equivalent :

i) I is an ideal in B.

ii) for all i, j ∈ I and b ∈ B , $I + j \in I$ and $b \leq i \Longrightarrow b \in I$.

SECTION-B(Short Answer Questions)

9. Answer any **three** questions. Each question carries **4** marks. **3 x 4=12 M**

a. Define the following terms and give one example for each:

a) Euler graph b) Hamiltonian graph.

b. Prove that there is one and only one path between every pair of vertices in a tree T
c. Define the following terms and give one example for each:
d. Planar graph b)Spanning tree
e. Prove that in a Boolean algebra B, $(x \wedge y)^1 = x^1 \vee y^1$.
f. Prove that every chain is a distributive lattice.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: IV

Paper - M 301: FUNCTIONAL ANALYSIS

Model Paper

Time: 3Hours Max.Marks:75

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **15** Marks. **4 x 15 = 60 M**

1. a) Define a Banach space. Prove that the real linear space R^n is a banach space .

b) In a Banach space B ,Prove that the vector addition and Scalar multiplication are jointly continuous

OR

2. **a)** State and prove Hahn-Banach theorem.
b) Prove that the mapping x$\rightarrow F_x: N \rightarrow N^{**}$ where $F_x(f)=f(x)$ $\forall f \in N^*$ is an isometric isomorphism of N into N^{**}

3. a) State and prove Open mapping theorem.
b) State and prove Uniform boundedness theorem

OR

4. a) If M and N are closed linear subspaces of a hillbert space H such that M ⊥ N, then prove that the linear subspace M+N is also closed.
b) State and prove Bessel's inequality(finite case).

5. a) Prove that the mapping y $\rightarrow f_y$ is a norm preserving mapping of H into H^* where $f_y(x) = \langle x, y \rangle$ for all x $\in H$.
b) If T is an operator on a Hillbert Space H for which $\langle Tx, x \rangle = 0$ for all x $\in H$. Then prove that T=0 on H.

OR

6. a) If T is an operator a Hillbert space H then prove that the following conditions are equivalent to one another.
1) $T^*T = I$
2) $\langle Tx, Ty \rangle = \langle x, y \rangle$ for all x and y
3) $\| Tx \| = \| x \|$ $for\ all\ x$.
b) Prove that a closed linear subspace M of H is invariant under T an operator T on H if and only if M^+ is invariant under T^*.

7. a) Prove that two matrices of A_n are similar if and only if they are the matrices of a single operator on H relative to (possibly) different bases.
   b) Prove that an operator T on H is singular if and only if $\in \sigma(T)$.

OR

8. State and prove Spectral theorem.

SECTION-B(Short Answer Questions)

9. Answer any THREE of the following questions.
   a) Prove that every normal linear space is a metric space.
   b) State and prove Schwartz inequality.
   c) Define an orthogonal set in a Hilbert Space H and give an example.
   d) Define eigen value and eigen vector.
   e) Let T be an operator on a Hilbert space H be such that the adjoint T^* of T is a polynomial in T, then operator T is normal.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics

Semester: III

Paper- M 302- LEBESGUE THEORY

Model Paper

Time: 3Hours Max.Marks:75

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **15** Marks. **4 x 15 = 60 M**

1. Define outer measure m* (A) a subset A of real numbers.
   Prove that the outer measure of an interval is its length.

(or)

2. Let f be an extened real valued function whose domine is measurable. Then prove that the following statements are equivalent.
   i. For each real number α,the set {x/f(x)≥ α} is measurable.
   ii. For each real number α,the set{x/f(x)≤ α} is measurable.
   iii. For each real number α,the set{x/f(x)<α} is measurable .
   iv. For each real number α,the set {x/f(x)>α} is measurable.
   v. For each real number α,the set {x/f(x)=α} is measurable.
3. State and prove Monotone convergence theorem.

(Or)

4. State and prove Lebesgue convergence theorem.
5. State Vitali lemma. Prove that a function f is of bounded variation on [a,b] if and only if f is the difference of two monotone real valued functions [a, b]

(Or)

6. a) Let f be an integer able function on[a, b] and suppose that F(x)=F(a)+$\int_a^x f(t)dt,$ then prove that
   F'(x)=f(x)[a] for almost all x in [a, b].
   b) Prove that if f is absolutely continuous on [a,b] then it is of bounded variation on[a, b].
7. Prove that a normal linear space X is complete if and only if every absolutely summable series is summable.

(Or)

8. Prove that the L^p spaces are complete.

SECTION-B(Short Answer Questions)

9. **Answer any Three question of the following.** $\mathbf{3\times 5 = 15\ M}$

a) If $A\subseteq B$ then prove that $m^*(A)\leq m^*(B)$.
b) If f is measurable and f=g almost everywhere then prove that g is measurable.
c) Define convergence in measure.
d) If $f\leq g$ almost everywhere on a set E of finite measure and f,g are bounded then prove that $\int_E f \leq \int_E g$.
e) State and prove Minkowski Inequality in L^p with $1\leq P<\infty$.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: III

Paper: M 303 – ANALYTICAL NUMBER THEORY

Model Paper

Time: 3Hours Max.Marks:75

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **15** Marks. **4 x 15 = 60 M**

1. a) Show that $\emptyset(m.n) = \emptyset(m)\emptyset(n)\ if$ (m,n)=1.
   b) For all f prove that $I * f = f * I = f$

OR

2. a) If n ≥ 1 prove that log n=$\sum_{d/n} \wedge(d)$.
   b) Let f be multiplicative. Then f is completely multiplicative if and only if $f^{-1}(n) = \mu(n)f(n)$ for all n ≥ 1.
3. a) State and prove Euler's summation formula.
   b) Show that the set of lattice points visible from the origin has density $\frac{6}{\pi^2}$.

OR

4. a) If x ≥ 2 , prove that log[x] != $x \log x - x + O(logx)$. Also prove that $\sum_{n\leq x} \wedge(n) \left[\frac{x}{n}\right] = x \log x - x + O(logx)$..
   b)For x >1 , prove that $\sum_{n\leq x} \emptyset(n) = \frac{3}{\pi^2}\, x^2 + O(xlogx)$
5. Show that the following relations are logically equivalent:
   a) $\lim_{x\to\infty} \frac{\pi(x)logx}{x} = 1$
   b) $\lim_{x\to\infty} \frac{\theta(x)}{x} = 1$
   c) $\lim_{x\to\infty} \frac{\varphi(x)}{x} = 1$

OR

6. a) For n ≥ 1 if P_n is the n^{th} prime show that P_n satisfies the inequalities $\frac{1}{6} n\ logn < P_n < 12\left(n\ logn + n\ log\frac{12}{e}\right)$
   b) Show that $\lim_{x\to\infty}\left(\frac{M(x)}{x} - \frac{H(x)}{x\ logx}\right) = 0$
7. a) If (a,m)=1, then show that the linear congruence $ax \equiv b\ (mod\ m)$ has exactly one solution.

b) State and prove Euler – Fermat theorem.

OR

8. a) State and prove Lagrange's theorem.

b) Show that if P is a prime ,all the coefficients of the polynomial

$f(x) = (x-1)(x-2) \dots .(x-p+1) - x^{p-1} + 1$ are divisible by p.

SECTION-B(Short Answer Questions)

9. Answer any **THREE** of the following. $3\times 5 = 15$

a) For n ≥ 1 show that $\varphi(n) = n^{\pi}_{p/n}\left(1 - \frac{1}{p}\right)$

b) If f is multiplicative ,then prove that f(1)=1.

c) For x >0 show that $0 \leq \frac{4(x)}{x} - \frac{\theta(x)}{x} \leq \frac{(\log x)^2}{2\sqrt{x}\, log^2}$.

d) Show that an integer n >0 is divisible by 9 if and only if the sum of its digits in decimal expansion is divisible by 9.

e) If the solving the congruence $5x \equiv 3(mod\ 24)$, is $x \equiv 15(mod\ 24)$ solve the congruence $25x \equiv 15(mod\ 120)$.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: III

Paper: M 304 - PARTIAL DIFFERENTIAL EQUATIONS

Model Paper

Time: 3Hours Max.Marks:75

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **15** Marks. **4 x 15 = 60 M**

1. Define Partial differential equation.If X is a vector such that X. Curl X=0 and μ is an arbitrary function of x,y,z then (μ, X). Curl(μ, X)=0

OR

2. Solve the Equation $\frac{dx}{y+\alpha z} = \frac{dy}{z+\beta x} = \frac{dz}{x+\gamma y}$.
3. A necessary and sufficient condition that a surface be an integral surface of PDE is that at each point its tangent element should touch the elementary cone of the equation.

OR

4. Find the complete integral of the PDE $(p^2+q^2)x=pz$ and deducethe solution which passes through the curve x=o, $z^3=4y$.
5. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^{2z}}{\partial y^2} = x - y$.

OR

6. Solve the equation $rq^2-2pqs+tp^2=pt-qs$.
7. State and prove Kelvin's Inversion theorem.

OR

8. Derive D-Alembert's solution of the one-dimensional wave equation.

SECTION-B(Short Answer Questions)

9. Answer any **THREE** of the following **$3\times 5 = 15$**
   a) Eliminate the arbitrary function 'f' from the equation z=x+y+f(xy).
   b) Find the general solution of the differential equation $x^2\frac{\partial z}{\partial x} - y^2\frac{\partial z}{\partial y}$ =(x+y)z .
   c) Find the complete integral of the equation $(p^2+q^2)y=qz$.
   d) Solve the equation $z(qs-pt)=pq^2$.
   e) Show that the surfaces $x^2+y^2+z^2=cx^{2/3}$ can form a family equipotential surface.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc. Mathematics Degree Examinations

Semester: III

Elective Paper:: M 305.1 – LATTICE THEORY

Model Paper

Time: 3 Hours Max.Marks:75

SECTION-A

(Essay Questions)

Answer **ALL** the questions. Each question carries **15** Marks. **4 x 15 = 60 M**

1. a) Prove that the partially ordered sets can be represented by the same diagram if and only if they are order isomorphic.

   b) State and prove kuratowsik – Zorn lemma.

OR

2. a) Show that in a partially ordered set of locally finite length bounded below, satisfying Jordan – Dedekind chain condition the dimension function can be defined.

   b) Let p be a poset satisfying minimum condition. Prove that for any $x \in$ p, their exist atleast one minimal element m in p such that x$\geq m$.

3. a) Prove that for any two elements a, b of the lattice L , $a \wedge b = b$ if and only if $b \vee a = a$.

   b) Prove that a lattice is a chain if and only if every one of its elements is meet – irreducible.

OR

4. Prove that two lattices are isomorphic if and only if they are order isomorphic.
5. a) Prove that every homomorphic image of a lattice bounded is likewise bounded

   b) Prove that every preserving mapping of a complete lattice into itself has a fixed element.

OR

6. a) Prove that every element of lattice satisfying the maximum condition is compact.

   b) Show that every uniquely complemented lattice is weakly complemented.

7. Prove that a lattice is modular if and only if no sub lattice of it is isomorphic with the lattice N_5 of M_5 .

OR

8. a) Show that every lattice is isomorphic to same sub lattice of a complete lattice.

b) Prove that any lattice in which every bounded non -void subset has an infimum is conditionally complete .

SECTION-B(Short Answer Questions)

9. Answer any **THREE** of the following questions: **$3\times 5 = 15$M**

a) Define a Partially ordered sets give an example.

b) Give an example of a poset of infinite length in which the length of the each such chain is finite.

c) Prove that every lattice has atmost one maximal element.

d) Show that every distributive lattice is modular.

e) Give an example of a distributive lattice which is not sectionally complemented.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: III

Elective Paper:: M 305.2 – COMMUTATIVE ALGEBRA

Model Paper

Time:3 Hours Max.Marks:75

SECTION-A (Essay Questions)

Answer **ALL** the questions. Each question carries **15** Marks. **4 x 15 = 60 M**

1. a) Every ring $A \neq 0$ has at least one maximal ideal, prove it ?

   b) Show that every non unit of A is continued in a maximal ideal.

(OR)

2. For ideal P and Q of a ring prove that

   a) $r(p) \supseteq p$

   b) $r(r(p)) = r(p)$

   c) $r(PQ) = r(P \cap Q) = r(P) \cap r(Q)$

   d) $r(P) = (1) \Leftrightarrow P = (1)$

   e) $r(P + Q) = r(r(p) + r(Q))$

   f) If P is prime then $r(R^n) = P$ for all $n > 0$.

3. a) If $L \supseteq M \supseteq N$ are A-Modules, then Show that $(L/N) / (M/N) \cong L/M$.

   b) If M_1, M_2 are sub modules of M , then show that $(M_1 + M_2) / M_1 \cong M_2 / (M_1 \cap M_2)$.

(OR)

4. a) Let M be a finitely generated A-Module, Let a be an ideal of A , and Let $\emptyset$ be an A – module endomorphism of M such that $\emptyset(M) \subseteq aM$. Then prove that $\emptyset$ satisfies an equation of the Form $\emptyset^n + a_1\emptyset^{n-1} + \ldots + a_n = 0$. Where a_i are in a.

   b) Suppose N is finitely generated as a B-module and that B is finitely generated as an A-module then prove that N is finitely generated as an A-module

5. a) let M be an M-module. Then show that the S^{-1} A nodules S^{-1} M and $S^{-1}A \otimes_A M$ are isomorphic.

   b) If S is a multiplicatively closed subset of a ring A, then show that the operation S^{-1} is exact.

(OR)

6. a) For any A-module M, then prove the following statements are equal:

i) M is a flat A-module.

ii) M_p is a flat A_p – module for each orime ideal P

iii) M_m is a flat A_m – module for each maximal ideal m.

b) Prove that if n is the nilradicalof A, then the nitradical of S^{-1} A is S^{-1} n.

7. a) Prove that Q be a primary ideal in a ring A. then r(q) be the smallest prime ideal containing q.

b) if r(a) is maximal then prove that a is primary.

(OR)

8. a) Let S be a multiplicatively closed subset of A, Q be a P-primary ideal .

i) if $S \cap P \neq \emptyset$, Show that $S^{-1}Q = S^{-1}A$.

ii) if , $S \cap P = \emptyset$ then show that $S^{-1}Q$ is $S^{-1}A$ primary and its contraction in A is Q.

b) State and prove second uniqness theorem.

SECTION-B(Short Answer Questions)

9. Answer any **THREE** of the following questions. **3× 5 =15 M**

a) Prove that the set η of all nilpotent elemenmt in a ring A an ideal and A/η has no nilpotent element $\neq 0$.

b) Prove that $x \in R \Leftrightarrow 1 - xy$ is a unit in A for all $y \in A$.

c) Show that M is a finitely generated A-module $\Leftrightarrow$ M is isomorphic to a quoitent of A^n for same integer $n > 0$.

d) Show that $S^{-1}A$ is a flat A- module.

e) Show that the primary ideals in z are (0) and (P^n) where P is prime.

GOVERNMENT AUTONOMOUS COLLEGE, RAJAMAHENDRAVARAM

M.Sc Mathematics Degree Examinations

Semester: III

Elective Paper:: M 305.3: COMPLEX ANALYSIS -II

Model Paper

Time:3 Hours Max.Marks:75

SECTION-A(Essay Questions)

Answer **ALL** the questions. Each question carries **15** Marks. **4 x 15 = 60 M**

1. a) State and prove maximum modulus theorem(first version)
   b) Prove that the function f : [a, b]→ ℝ is convex if and only if the set
   A = { (x,y)/a≤ x≤b, f(x) ≤y} is convex.

OR

2. State and prove Phragmen – Lindelof theorem.
3. a) Define the term – normal. Prove that a subset $\mathcal{F} \leq C(G, \Omega)$ is normal if and only if $\mathcal{F}$ is compact.
   b) State and prove Weienstron Factorization theorem.

OR

4. a) Define the infinite product of complex numbers.
   Prove that $\prod_{n=2}^{\infty}(1-\frac{1}{n^2}) = \frac{1}{2}$.
   b) If $|z| \leq 1$ and p ≥ 0, prove that $|1-E_p(z)| \leq |z|^{p+1}$.
5. a) State and prove Mittag –Leffler's theorem.

OR

6. State and prove Schwartz Reflextion principle
7. a) State and prove Mean Value theorem.
   b) State and prove Harnack's theorem.

OR

8. State and prove Hadamard's Factorization theorem.

SECTION-B(Short Answer Questions)

9. Answer any **THREE** of the following. 3 x 5 = 15 M
   a) Show that if f : (a, b) ⟶ R is convex then f is continuous .
   b) Define the absolute convergence of an infinite product $\prod z_n$.
   c) For $|z| < 1$, prove that $(1+z)(1+z^2)(1+z^4)(1+z^8)\ldots = \frac{1}{1-z}$.
   d) State Montel's theorem.
   e) Define genus of an entire function f.