1/3/2021
1/2/2025

Practical - 1

Classification & Tabulation

Problem No. :- 1

In a survey of 50 families in a village, the No. of children per family was recorded and the following data obtained.

1, 0, 2, 3, 4, 5, 6, 7, 2, 3
4, 0, 2, 5, 8, 4, 5, 12, 6, 3
2, 7, 6, 5, 3, 3, 7, 8, 9, 7
9, 4, 5, 4, 3, 10, 11, 9, 8, 1,
0, 2, 10, 11, 10, 12, 0, 7, 8, 9

Represent the data in the form of discrete frequency Distribution.

Problem No. :- 2

Prepare a frequency distribution of the Marks obtained out of 100 for the following students (continuous)

15, 45, 40, 42, 50, 60, 62, 68, 70, 42, 75, 75, 80, 81,
25, 26, 31, 32, 78, 45, 31, 45, 42, 43, 55, 56, 78, 80,
81, 62, 60, 62, 58, 69, 70, 45, 50, 56, 72, 58, 75, 62,
62, 60, 70, 35, 37, 40, 55.

Problem No. :- 3

Point out the mistakes in the following table drawn to show the distribution of population according to sex, age & literacy

Gender	0 - 25	25 - 50	50 - 75	75 - 100
Males Females				

Problem No. :- 4

In a certain data, the following main four characteristics with their sub characteristics are present

Main characteristics	Sub characteristics
Locality	Urban, rural
Religion	Hindus, Non-Hindus
Gender	Males, Females
Age	0-30, 30-60, 60 above

Prepare a suitable table for the following data.

1/3/2021 **Practical-1**

Problem No.:-1

Aim :- To present the data in the form of discrete frequency Distribution

Formula & procedure :-

1) Analyze the given data and find frequency by using tally Marks.
2) Represent the Analyzed data by using tabulation.

Discrete frequency Distribution :-

If the data series are presented in such a way that indicating its exact measurement of units then it is called as discrete frequency Distribution.

Tabulation of Data :-

The logical and systematic organisation of statistical data in rows and columns.

Calculation :-

Table-1 :-

No. of children of families in a village

No. of children of 50 families.

No. of children	Tally	frequency
0	IIII	4
1	II	2
2	~~IIII~~	5
3	~~IIII~~ I	6
4	~~IIII~~	5
5	~~IIII~~	5
6	III	3
7	~~IIII~~	5
8	IIII	4
9	IIII	4
10	III	3
11	II	2
12	II	2

Inference :- The discrete frequency Distribution is obtained

Problem No. :-2

Aim :- To prepare frequency Distribution of the Marks obtained out of 100 by 50 students

Formula & procedure :-

1) Let class intervals which of Equal size and prepare frequency distribution by using tally Marks.

Continuous frequency Distribution

Continuous data series is one where the measurements are only approximations and are expressed in class intervals within certain limits.

Tabulation of data :-

The logical and systematic organisation of statistical data in rows and columns

Calculation :-

Table No. :- 2

Frequency Distribution of Marks obtained by 50 students

class interval	Tally	Frequency
10-20	I	1
20-30	II	2
30-40	IIII	4
40-50	~~IIII~~ IIII	9
50-60	~~IIII~~ III	8
60-70	~~IIII~~ ~~IIII~~ I	11
70-80	~~IIII~~ IIII	9
80-90	IIII	4

Inference :- The frequency distribution of Marks out of 100 obtained by 50 students is obtained.

Problem No.:-3

Aim :- To point out the mistakes in given table and form a good table.

Formula & procedure :-

Tabulation of data :-

The logical and systematic organisation of statistical data in rows and columns.

we plot the table by using following parts

1) Table No.
2) Title
3) Head Note
4) caption
5) stubs
6) Body of the table
7) Foot Note
8) source Note.

Calculation :-

Table No.:-4

No. of population according to sex, age & literacy.

Age Group	Gender			literacy		
	M	F	T	M	F	T
0-25						
25-50						
50-75						
75-100						

Inference :- We point out mistakes in given table and correct them then drawn a proper table for given information.

Problem No:- 4

Aim :- To present certain data including main four characteristics with their sub - characteristics

Formula & Procedure

Tabulation of data :-

The logical and systematic organisation of statistical data in rows and columns we plot the table by using following parts

1) Table No.
2) Head Note
3) caption
4) skubs
5) body of table
6) foot Note
7) Source Note

Calculation

Table No. 4

classifying Table according to characteristics

Age	Locality		Religion		Gender	
	urban	Rural	Hindu	Non-Hindu	Male	Female
0 - 30						
30 - 60						
Above 60						

Inference :- Drawn a table according to Non characteristics including with their sub-characteristics.

good

8/3/2021

Practical - 2

Diagrammatic Representation of Data

Problem No. :- 1

A survey shows the following results
Who bribes Whom

Sector	Percentage
Companies to tax authorities	10.0
Employees to Employees	4.8
Company to Employees	12.8
company to Government	40.0
company to company	19.7
company to customers	12.7

Represent the following information in pie chart

Problem No. :- 2

Draw a multiple bar diagram from the following data.

Year	Sales	Gross profit (₹ 1000's)	Net profit (₹ 1000)
2016	120	40	20
2017	135	45	30
2018	140	55	35
2019	150	60	40

Problem No. :-3

Draw a Histogram and frequency polygon from the following data.

Marks	No. of students
0-10	4
10-20	6
20-30	14
30-40	16
40-50	14
50-60	8
60-70	16
70-80	12
80-90	13
90-100	5

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8/3/2021 Practical-2

Problem : 1

Aim :- To represent the following information in piechart.

Formula & procedure :-

Pie-diagram or pie chart

A pie diagram is also a component diagram but it as a circle whose area is proport-ionally divided among the components it represents. Formula for finding Angular or degree of the component by using percentages is :

Angular component of circle = percentage figure × 3.6°

=

Calculation :-

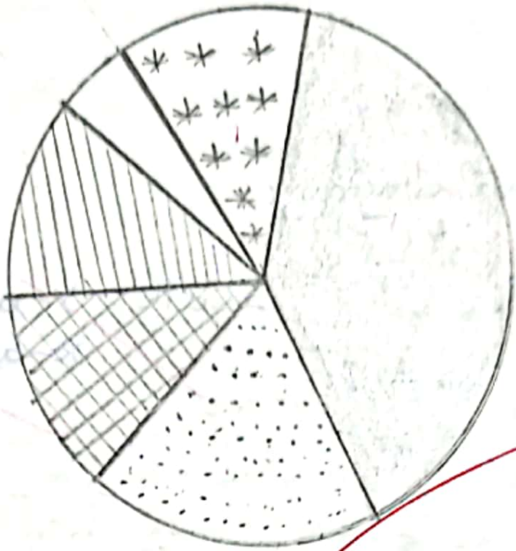
Table - 1

Sector	Percenta	Degrees
companies to tax autho	10.0	$10 \times 3.6° = 36°$
Employees to Employees	4.8	$4.8 \times 3.6° = 17.28°$
Company to Employees	12.8	$12.8 \times 3.6 = 46.08°$
company to Govt	40.0	$40 \times 3.6° = 144°$
company to company	19.7	$19.7 \times 3.6° = 70.92°$
company to customers	12.7	$12.7 \times 3.6° = 45.72°$

Diagram :- 1

Pie-chart

?



Pie chart showing who bribes whom

- companies to tax authorities
- Employees to Employees
- company to Employees
- company to Govt
- company to company
- company to customers

Inference :- A piechart is drawn to represent the given information who bribes whom.

Problem No.:- 2

Aim :- To present the given information in a Multiple bar diagram

Formula & procedure :-

Multiple bar diagram

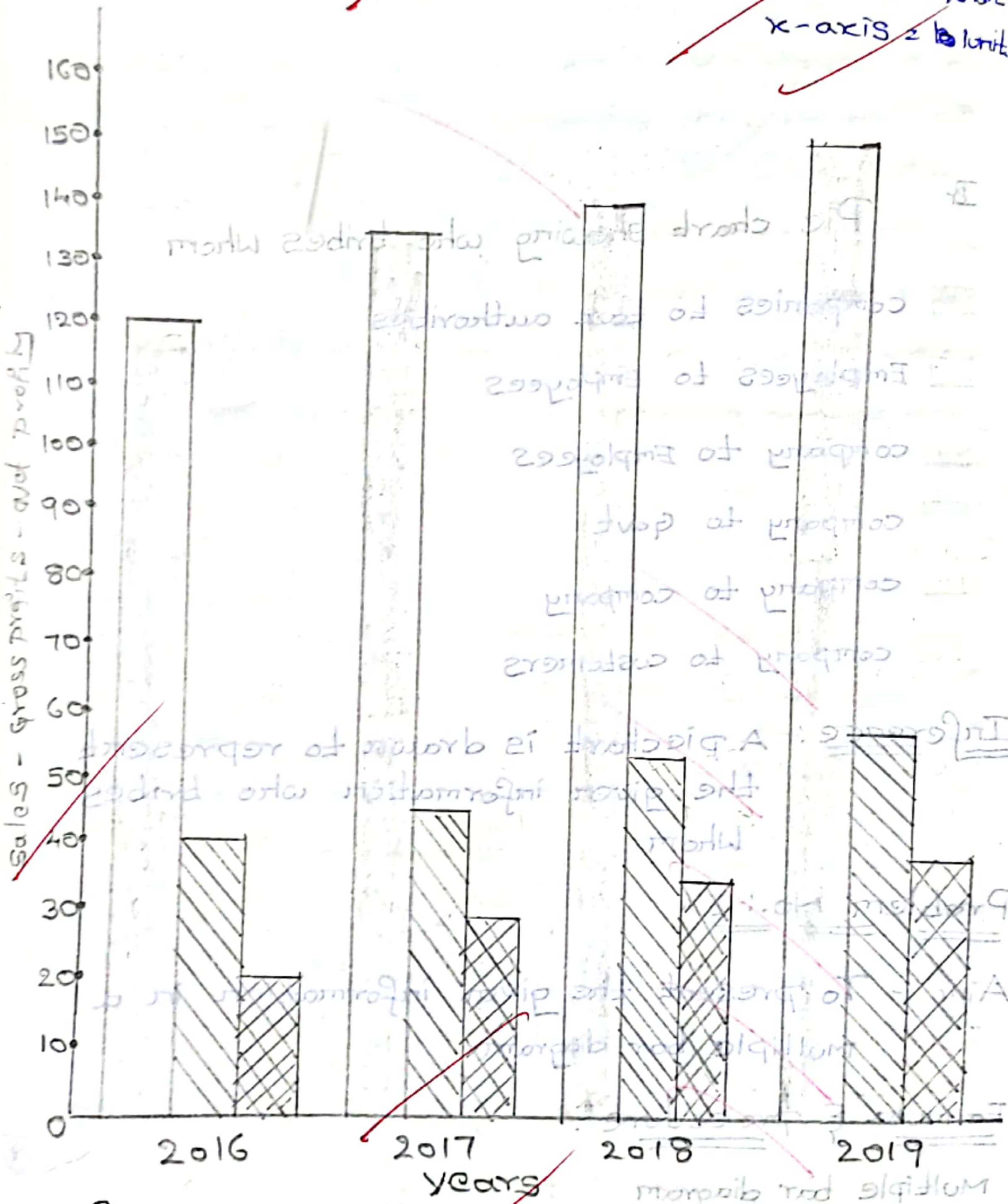
It is a diagram depicting two or more characteristics in the form of adjacently placed bars of height proportion to the magnitude of characteristics.

Calculation :-

Diagram - 2

Multiple bar diagram

Scale :- Y-axis = 1unit = 10 km
x-axis = 1unit = 1 yr



Inference :- A multiple bar diagram is drawn to show sales, Gross profit, net profit in 2016, 2017, 2018, 2019 years respectively.

Problem No. :- 3

Aim :- To draw a Histogram and Frequency polygon for following data.

Formula & Procedure :-

Histogram :-

It is a graph or a frequency distribution consisting of rectangles in which the C.I are plotted along the x-axis and their respective frequencies on y-axis

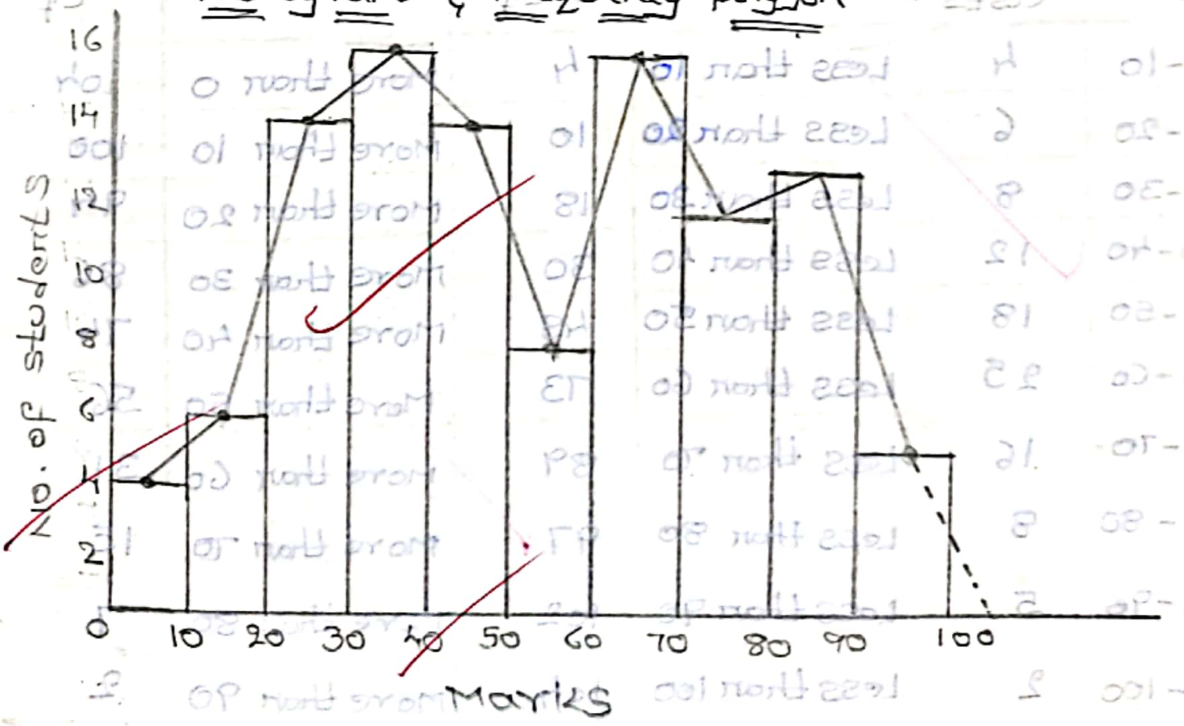
Frequency Polygon :-

A frequency polygon is a plane closed figure bounded by straight lines used for depicting frequency data.

A frequency polygon is obtained by Histogram by Joining the midpoints of the topside of the consecutive rectangles of the Histogram using straight lines.

Calculation :- Diagram-3 ; Scale :- x-axis 1cm = 10, y-axis 1cm = 2 students

Histogram & Frequency polygon



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Inference :- A Histogram is obtained to show Marks scored by No. of students

15/03/2021

Practical-3

Measures of central tendency

Problem No.:-1

Calculate A.M for the following data

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of students	5	10	25	30	20	10

Problem No.:-2

Consider the following Distribution

x	0-10	10-20	20-30	30-40	40-50
f	12	18	20	25	23

Compute Mean, Median and Mode.

Problem No.:-3

Calculate Mean, Median and Mode for the ungrouped Data

2, 3, 2, 4, 4, 5, 2, 7, 2, 8

Problem No.:-4

With the help of the given data find

P_{40}, Q_6, , Q_2, Q_3

Age (in years)	10-14	15-19	20-24	25-29	30-34	35-39
No. of persons	5	10	15	20	10	5

15/3/21 **Practical - 3**

Problem No. :- 1

Aim :- To find A.M for following Data

Formula & procedure :-

Direct Method $\bar{x} = \frac{\sum_{i=1}^{n} fm}{\sum_{i=1}^{n} f}$

$\sum fm$ = Sum of mid pt. of classes Multiplied by their respective class frequencies

$\sum f$ = Total No. of observations

Assumed Mean $\bar{x} = A + \frac{\sum_{i=1}^{n} fd}{\sum_{i=1}^{n} f}$

$d = (m - A)$ and m is mid pt of respective class

A = Assumed Mean & $\sum_{i=1}^{n} f = N$ = Total No. obs.

Step Deviation Method $\bar{x} = A + \frac{\sum fd'}{\sum f} \times C$

$d' = \frac{m - A}{C}$; m is Mid pt. of respective class & c is common factor in d

A = Assumed Mean

$\sum f = N$ = Total No. of observations.

Arithmetic Mean defined as sum of obs. divided by No. of obs.

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_N}{N}$$

Calculation :-

Marks (x)	No. of students (f)	Mid value (M)	fm	d=m-35	fd	d'=d/10	fd'
0-10	5	5	25	-30	-150	-3	-15
10-20	10	15	150	-20	-200	-2	-20
20-30	25	25	625	-10	-250	-1	-25
30-40	30	35(A)	1050	0	0	0	0
40-50	20	45	900	10	200	1	20
50-60	10	55	550	20	200	2	20
	$\Sigma f = 100$		$\Sigma fm = 3300$		$\Sigma fd = -200$		$\Sigma fd' = -20$

Direct Method $\bar{x} = \frac{\Sigma fm}{\Sigma f} = \frac{3300}{100} = 33$

Assumed Mean $\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$

$= 35 + \left(\frac{-200}{100}\right) = 33$

Step-Deviation Method $\bar{x} = A + \frac{\Sigma fd'}{\Sigma f} \times C$

$= 35 + \left(\frac{-20}{100}\right) \times 10$

$= 33$

Inference :- We get Arithmetic Mean of students is 33 by using three methods.

Problem No. :- 2

Aim :- To find Mean, Median and Mode for given Data.

Formula & Procedure :-

Mean :- Sum of obs. divided by No. of obs.

Step Deviation $\bar{x} = A + \frac{\Sigma fd'}{\Sigma f} \times c$

$d' = \frac{m-A}{c}$; m is Mid pt. of respective class & c is length of class

A = Assumed Mean ; $\Sigma f = N$ = Total No. of obs.

Median :- Middle value in the Data set when its Elements are arranged in sequential order.

$$\text{median} = L + \frac{N/2 - c.f}{f} \times h$$

L = Lower limit of Median class

f = frequency of Median class

c.f = c.f of class preceding the median Data

h = length of Median class interval.

Mode :- Most occuring value in given series

$$\text{Mode} = L + \frac{f_s}{f_p + f_s} \times c$$

L = Lower limit of Model class

f_p = frequency of class preceding modal class

f_s = frequency of class succeeding Modal class.

c = class interval

<u>Calculation</u> :-

x	f	m	$d' = \frac{m-35}{10}$	fd'	c.f
0 - 10	12	5	$-30/10 = -3$	-36	12
10 - 20	18	15	$-20/10 = -2$	-36	30 (c.f)
20 - 30	20 f_p	25(f)	$-10/10 = -1$	-20	50
30 - 40	25 f	35(A)	0	0	75
40 - 50	23 f_s	45	$10/10 = 1$	23	98

$\Sigma f = 98$ $\Sigma fd' = -69$

$N/2 = 98/2 = 49^{th}$ term

$$\text{Mean} = A + \frac{\Sigma fd'}{\Sigma f} \times c$$

$$= 35 + \frac{(-69)}{\cancel{98}_{49}} \times \cancel{10}^{5}$$

$$= 35 + \left(\frac{-345}{49}\right)$$

$$= 35 - 7.04$$

$$= 27.96$$

$$\text{Median} = L + \frac{N/2 - c.f}{f} \times h$$

$$= 20 + \frac{49 - 30}{\cancel{25}\,5} \times \cancel{10}\,2$$

$$= 20 + \frac{38}{5}$$

$$= 20 + 7.6$$

$$= 27.6.$$

$$\text{Mode} = L + \frac{f_s}{f_p + f_s} \times c$$

$$= 30 + \frac{23}{23 + 20} \times 10$$

$$= 30 + \frac{230}{43} = 30 + 5.34 = 35.34$$

Inference :- We get Mean, median & Mode for given Data.

Problem No. :- 3

Aim :- To calculate mean, median & mode for ungrouped data.

Formula & procedure :-

Mean :- Sum of observations divided by No. of observations.

Median :- Middle value of given data when obs. are arranged in sequential order.

Mode :- Most frequent value in given set

Calculation :-

$$\text{Mean} = \frac{\text{sum of observations}}{\text{No. of observations}}$$

$$= \frac{2+3+2+4+4+5+2+7+2+8}{10}$$

$$\therefore \text{Mean} = \frac{39}{10} = 3.9$$

Median =

Arrange the data in Ascending order

2, 2, 2, 2, 3, 4, 4, 5, 7, 8

Here $n = 10$ (even)

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ item}$$

$$= \left(\frac{10+1}{2}\right)^{th} \text{ item} = 5.5^{th} \text{ item}$$

$$\Rightarrow \frac{5^{th}\text{ item} + 6^{th}\text{ item}}{2} = \frac{3+4}{2} = 3.5$$

Median is 3.5

$\therefore$ Mode is 2 (Most occuring value in given data)

Inference :- We got the values of mean, Median and Mode for given ungrouped data

Problem No. 1-4

Aim :- To find P_{40}, D_6, Q_2, & O_3 values

Formula & Procedure :-

Percentile :- The percentile values divide the distribution into 100 parts.

$$P_x = L + \left[\frac{x \cdot n/100 - c.f}{f} \times C\right]$$

Decile :- The decile values divide the distribution into 10 parts.

$$D_x = L + \left[\frac{x \cdot n/10 - c.f}{f}\right] \times C$$

Quartile :- The quartile values divide the distribution into 4 parts

$$Q_x = L + \left[\frac{x \cdot n/4 - c.f}{f}\right] \times C$$

Octile :- The octile values divide the distribution into 8 parts.

$$O_x = L + \left[\frac{x \cdot n/4 - c.f}{f}\right] \times C$$

Where ; L = Lower limit of the percentile class which contains the x^{th} value

c.f = cumulative frequency upto k

f = frequency of class

c = class interval

N = Total No. of observations

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Calculation :-

Age	No. of Persons	c.f
10-14	5	5
15-19	10	15
20-24	15	30
25-29	20	50
30-34	10	60
35-39	5	65

Percentile :-

For P_{40} first find out, $\frac{x \cdot n}{100} = \frac{\cancel{40}^{2} \times \cancel{65}^{13}}{\cancel{100}_{5}} = 26$

The value 26 lies b/w 15 & 30. Therefore the percentile class is 20-24

Hence; $P_{40} = l + \left[\frac{x \cdot n/_{100} - c \cdot f}{f} \times c\right]$

$= 20 + \frac{26-15}{15} \times 4$

$= 20 + \frac{44}{15} = 20 + 2.93 = 22.93$

Decile :-

For D_6 first find out $\frac{x \cdot n}{10} = \frac{\cancel{6}^{3} \times \cancel{65}^{13}}{\cancel{10}_{5}} = 39$

The value 39 lies b/w 30 & 50. Therefore the decile class is 25-29.

Hence; $D_6 = L + \left[\frac{x \cdot n/_{10} - c \cdot f}{f} \times c\right]$

$$= 25 + \frac{39-30}{\cancel{20}\ 5} \times \cancel{4}\ 1$$

$$= 25 + 9/5 = 1.8$$

$$= 26.8$$

Quartile :-

For Q_2 first find out $\frac{x \cdot n}{4} = \frac{2 \times 65}{\cancel{4}\ 2} = 32.5$

The value 32.5 lies b/w 30 & 50. Therefore the quartile class is 25 – 29

Hence $Q_2 = L + \left[\frac{x \cdot n/4 - c.f}{f} \times c\right]$

$$= 25 + \left[\frac{32.5 - 30}{\cancel{20}\ 5}\right] \times \cancel{4}$$

$$= 25 + \frac{2.5}{5} = 0.5 + 25 = 25.5$$

Octile :-

For O_3 first find out $\frac{x \cdot n}{8} = \frac{3 \times 65}{8} = 24.375$

The value 24.375 lies b/w 15 and 30. Therefore the octile class is 20 – 24

Hence $O_3 = l + \left[\frac{x \cdot n/8 - c.f}{f} \times c\right]$

$$= 20 + \frac{24.375 - 15}{15} \times 4$$

$$= 20 + \frac{9.375}{15} \times 4$$

$$= 20 + \frac{37.5}{15} = 20 + 2.5 = 22.5$$

Inference :- Hence P_{40}, D_6, Q_2 & O_3 values obtained.

22/03/21

Practical-4

Measures of Dispersion

Problem No. :-1

compute coefficient of Q.D from the following Data.

Marks	10	20	30	40	50	60
No. of students	4	7	15	8	7	2

Problem No. :-2

Find Mean Deviation of the following Data

Size	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	7	12	18	25	16	14	8

Problem No. :-3

Find the S.D from the following Data

Age	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of persons dying	15	15	23	22	25	10	5	10

22/03/21

Practical - 4

Problem No. :- 1

Aim :- To compute coefficient of quartile deviation.

Formula & Procedure :-

coefficient of quartile Deviation

A relative measure of Dispersion based on the quartile deviation is called the coefficient of quartile deviation. It is defined as coefficient of quartile deviation $= \frac{Q_3 - Q_1/2}{Q_3 + Q_1/2} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$

Where Q_1 is lower quartile & Q_3 is upper quartile

$\therefore Q_r = L + \left[\frac{x \cdot n/4 - c.f}{f} \times c\right]$ or $Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term

$Q_3 = 3\left[\frac{n+1}{4}\right]^{th}$ term

Calculation :-

Marks	No. of students	c.f
10	4	4
20	7	11
30	15	26
40	8	34
50	7	41
60	2	43

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} \text{ term}$$

$$= \left(\frac{43+1}{4}\right)^{th} \text{ term} = \left(\frac{44}{4}\right)^{th} \text{ term} = 11^{th} \text{ term}$$

$$Q_3 = 3\left[\frac{n+1}{4}\right]^{th} \text{ term}$$

$$= 3\,(11)^{th} \text{ term} = 33^{th} \text{ term} = 40$$

∴ Hence coefficient of Q.D $= \dfrac{Q_3 - Q_1}{Q_3 + Q_1}$

$$= \frac{40-20}{40+20}$$

$$= \frac{1}{3}$$

Inference :- coefficient of Q.D is $\frac{1}{3}$

Problem No. :- 2

Aim :- To compute mean deviation of the following data.

Formula & Procedure :-

Mean deviation of a series is the arithmetic average of the deviation of various items from a measure of central tendency (mean, median & mode)

$$\text{coefficient of M.D} = \frac{\text{M.D}}{\text{Mean/Median/Mode}}$$

Where Mean $\bar{x} = A + \dfrac{\Sigma fd'}{N} \times c$

Median = size of $\left(\frac{N+1}{2}\right)^{th}$ term

Calculation :-

size x	frequency f	Mid pt M	$d' = \frac{m-35}{10}$	fd'	$\lvert D \rvert = m-35.5$	$f\lvert D \rvert$
0 - 10	7	5	-3	-21	30.5	213.5
10 - 20	12	15	-2	-24	20.5	246
20 - 30	18	25	-1	-18	10.5	189
30 - 40	25	35(A)	0	0	0.5	12.5
40 - 50	16	45	1	16	9.5	152
50 - 60	14	55	2	28	19.5	273
60 - 70	8	65	3	24	29.5	236
	$\Sigma f = 100$			$\Sigma fd' = 5$		$\Sigma f\lvert D \rvert = 1322$

$$\bar{x} = A + \frac{\Sigma fd'}{N} \times c$$

$$= 35 + \frac{\cancel{5}}{\cancel{100}_{\cancel{10}\,2}} \times \cancel{10} = 35 + 0.5 = 35.5$$

$$\text{Mean Deviation} = \frac{\Sigma f\lvert D \rvert}{N} = \frac{1322}{100} = 13.22$$

$$\text{Coefficient of M.D} = \frac{\text{M.D}}{\text{Mean}}$$

$$= \frac{13.22}{35.5} = 0.372$$

Inference :- Coefficient of Mean deviation is 0.372. M.d is 13.22.

Problem NO. :- 3

Aim :- To find standard deviation for following data

Formula & Procedure :-

standard deviation measures the absolute dispersion or variability of a distribution

If x_i/f_i, $i = 1, 2, ---n$ is the grouped frequency distribution then the standard deviation is given by

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^2}$$

Where $N = \sum_{i=1}^{n} f_i$

Calculation :-

Age	No. of persons (f)	Mid value (x_i)	$f_i x_i$	$f_i (x_i - \bar{x})^2$ $= f_i (x_i - 35.16)^2$	$f_i x_i^2$
0-10	15	5	75	13644.38	375
10-20	15	15	225	6096.38	3375
20-30	23	25	575	2374.18	14375
30-40	22	35	770	0.5632	26950
40-50	25	45	1125	2420.64	50625
50-60	10	55	550	3936.25	30250
60-70	5	65	325	4452.12	21125
70-80	10	75	750	15872.25	56250
	$\Sigma f = 125$		$\sum_{i=1}^{n} f_i x_i = 4395$	$\sum_{i=1}^{n} f_i (x_i - \bar{x})^2 = 48796.76$	$\sum_{i=1}^{n} f_i x_i^2 = 203325$

$\therefore \quad N = \sum_{i=1}^{n} f_i = 125$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{n} f_i x_i = \frac{4395}{125} = 35.16$$

$$S.D = \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^2}$$

$$= \sqrt{\frac{48796.76}{125}}$$

$$= \sqrt{390.37408}$$

$$= 19.75$$

Alternative Methode :-

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{n} f_i x_i^2 - (\bar{x})^2}$$

$$= \sqrt{\frac{203325}{125} - (35.16)^2}$$

$$= \sqrt{1626.6 - 1236.2256}$$

$$= \sqrt{390.3744}$$

$$= 19.75$$

Inference :- S.D. of given data is 19.75

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31-3-21

Practical-5

Moments, skewness and kurtosis

Problem :- 1

The first four moments of a distribution about value 5 are -4, 22, -117 and 560. Find four central moments. Also find β_1 and β_2.

Problem :- 2

Find first four moments about mean also calculate sheppard's corrections for following data.

x_i	0	1	2	3	4	5	6	7	8
f_i	1	8	28	56	70	56	28	8	1

Problem :- 3

Find karl pearson's coefficient of skewness to the following data.

C.I	0-2	2-4	4-6	6-8	8-10	10-12	12-14
f	6	8	17	21	15	11	2

Problem :- 4

calculate Bowley's coefficient of skewness to the following data.

C.I	5-10	10-15	15-20	20-25	25-30	30-35	35-40
f	45	26	16	13	12	2	4

Problem :- 1

Aim :- To find four central moments μ_1, μ_2, μ_3 & μ_4 and also β_1 & β_2

Formula & Procedure :-

Central moments :- The r^{th} moment about the mean $\bar{x}$ are called central moments central moments are denoted by μ_r.

$$\boxed{\mu_r = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \bar{x})^r}$$

Non-central Moments :- The r^{th} moment about any point 'A' are called Non-central moments. Non-central moments are deno-ted by μ_r'.

$$\boxed{\mu_r' = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^r}$$

The first four Moments; $\mu_1 = 0$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

Skewness :- It means lack of symmetry. It gives an idea about the shape of the curve of given data.

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} ; \quad \beta_2 = \frac{\mu_4}{\mu_2^2}$$

Calculation :-

In the given problem ; given first four N.C.M

$\therefore \mu_1' = -4$; $\mu_2' = 22$

$\mu_3' = -117$; $\mu_4' = 560$

$\therefore$ We can find the four central Moments

$\therefore \boxed{\mu_1 = 0}$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 22 - (-4)^2$$

$$= 22 - 16$$

$$\boxed{\mu_2 = 6}$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$$

$$= (-117) - 3(-4)(22) + 2(-4)^3$$

$$= -117 + 264 - 128$$

$\boxed{\cancel{\mu_3 = 253}\ 19}$ $\therefore \boxed{\mu_3 = 19}$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 560 - 4(-117)(-4) + 6(22)(-4)^2 - 3(-4)^4$$

$$= 560 - 1872 + 2\cancel{464}112 - 768$$

$\boxed{\cancel{\mu_4 = 4128}}$ $\therefore \boxed{\mu_4 = 32}$

We can Also find β_1 & β_2

$$\therefore \beta_1 = \frac{\mu_3^2}{\mu_2^3}$$

$$= \frac{(19)^2}{(6)^3} = \frac{361}{216} = 1.671$$

$$\therefore \boxed{\beta_1 = 1.671}$$

$$\therefore \beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{32}{(6)^2} = \frac{\cancel{32}^{\,8}}{\cancel{36}_{\,9}} = \frac{8}{9} = 0.89$$

$$\therefore \boxed{\beta_2 = 0.89}$$

Inference :- The four central Moments are $\mu_1, \mu_2, \mu_3, \& \mu_4$ is $0, 6, 19 \& 32$ and also skewness corrections are $\beta_1 \& \beta_2$ is $1.671 \& 0.89$

Problem :- 2

Aim :- To find four moments about Mean and also find sheppard's correction.

Formula & Procedure :-

Central Moments :- The r^{th} moment about the Mean $\bar{x}$ are called central Moments. It is Denoted by μ_r.

$$\boxed{\mu_r = \frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \bar{x})^r}$$

Non-central Moments :- The r^{th} Moment about any point 'A' are called Non-central Moments. It is Denoted by μ_r'

$$\boxed{\mu_r' = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - A)^r}$$

The first Moments from the C.M & NCM are:

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

Here, $\mu_1' = \frac{1}{N}\sum_{i=1}^{n} f_i x_i \times h^1$

$$\mu_2' = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^2 \times h^2$$

$$\mu_3' = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^3 \times h^3$$

$$\mu_4' = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^4 \times h^4$$

Sheppard's Correction :-

In statistics, sheppard's corrections are approximate corrections to Estimates of Moments computed from binned data.

$$\therefore \mu_2 = \mu_2 - \frac{h^2}{12}$$

$$\mu_4 = \boxed{\mu_4 - \frac{1}{2}h^2\mu_2 + \frac{7}{240}\cdot h^4}$$

Calculation :- Table :- 1

x_i	f_i	$f_i x_i$	$f_i x_i^2$	$f_i x_i^3$	$f_i x_i^4$
0	1	0	0	0	0
1	8	8	8	8	8
2	28	56	112	224	448
3	56	168	504	1512	4536
4	70	280	1120	4480	17920
5	56	280	1400	7000	35000
6	28	168	1008	6048	36288
7	8	56	392	2744	19208
8	1	8	64	512	4096
	$\Sigma f =$ 256	$\Sigma f_i x_i =$ 1024	$\Sigma f_i x_i^2 =$ 4608	$\Sigma f_i x_i^3 =$ 22528	$\Sigma f_i x_i^4 =$ 117504

$$\therefore \mu_1' = \frac{1}{N} \sum_{i=1}^{n} f_i x_i \times h$$

$$= \frac{1}{256} [1024 \times 1]$$

$$\boxed{\mu_1' = 4}$$

$$\therefore \mu_2' = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^2 \times h^2$$

$$= \frac{1}{256} [4608 \times (1)^2]$$

$$= \frac{4608}{256} = 18$$

$$\boxed{\therefore \mu_2' = 18}$$

$$\therefore \mu_3' = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^3 \times h^3$$

$$= \frac{1}{256} [22528 \times (1)^3]$$

$$= \frac{22528}{256} = 88$$

$$\boxed{\therefore \mu_3' = 88}$$

$$\mu_4' = \frac{1}{N} \sum_{i=1}^{n} f_i x_i^4 \times h^4$$

$$= \frac{1}{256} [117504 \times (1)^4]$$

$$= \frac{117504}{256} = 459$$

$$\boxed{\therefore \mu_4' = 459}$$

∴ The first four Moments are :-

$$\boxed{\therefore \mu_1' = 0}$$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$= 18 - (4)^2 = 18 - 16 = 2$$

$$\boxed{\therefore \mu_2 = 2}$$

$$\mu_3 = \mu_3' - 3\mu_1'\mu_2' + 2\mu_1'^3$$

$$= 88 - 3(4)(18) + 2(4)^3$$

$$= 88 - 216 + 128 = 0$$

$$\boxed{\therefore \mu_3 = 0}$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$$

$$= 459 - 4(88)4 + 6(18)(4)^2 - 3(4)^4$$

$$= 459 - 1408 + 1728 - 768$$

$$= 11$$

$$\boxed{\therefore \mu_4 = 11}$$

∴ Sheppard's correction :-

$$\therefore \mu_2 = \mu_2 - \frac{h^2}{12}$$

$$= 2 - \frac{1}{12}$$

$$= \frac{23}{12} = 1.9167$$

$$\boxed{\therefore \mu_2 = 1.9167}$$

$$\mu_4 = \mu_4 - \frac{1}{2}h^2\mu_2 + \frac{7}{240}h^4$$

$$= 11 - \frac{1}{\cancel{2}}(1)^2\cancel{2} + \frac{7}{240}(1)^4$$

$$= 10 + 0.029$$

$$\boxed{\mu_4 = 10.029}$$

Inference :- The four central moments are 0, 2, 0, 11 and Sheppard's correction :- 1.9167 & 10.029.

Problem :- 3

Aim :- To find karl pearson's coefficient of skewness.

Formula & procedure :-

Karl pearson's coefficient of skewness

This Method is Most frequently used for measuring skewness. The formula for measuring coefficient of skewness is given by.

$$K.P.S_k = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

Here, **skewness** :- It means lack of symmetry. It gives an idea about the shape of the curve of given data.

From the standard Deviation :-

$$\boxed{\sigma = h \cdot \sqrt{\frac{1}{N} \sum_{i=1}^{n} f_i d_i^2 - \left(\frac{1}{N} \sum_{i=1}^{n} f_i d_i\right)^2}}$$

Mean $(\bar{x}) = A + \frac{\sum_{i=1}^{n} f_i d_i}{N} \times h$

Mode $(M) = l + \left[\frac{f_s}{f_p + f_s}\right] \times h$

Calculation :- Table :- 1

class Interval (ci)	Frequency (f)	Mid value (x_i)	$d_i = \frac{x_i - A}{h}$	$f_i d_i$	$f_i d_i^2$
0 – 2	6	1	−3	−18	54
2 – 4	8	3	−2	−16	32
4 – 6	17 (f_p)	5	−1	−17	17
l (6) – 8	21 (f)	7 (a)	0	0	0
8 – 10	15 (f_s)	9	1	15	15
10 – 12	11	11	2	22	44
12 – 14	2	13	3	6	18
	$\Sigma f = 80$			$\Sigma f_i d_i = -8$	$\Sigma f_i d_i^2 = 180$

Here ; $h = 2$

$$\therefore \text{Mean}(\bar{x}) = A + \sum_{i=1}^{n} \frac{f_i d_i}{N} \times h$$

$$\bar{x} = 7 + \left(\frac{-\cancel{8}}{\cancel{80}_{\cancel{40}\,5}}\right) \times \cancel{2}$$

$$= 7 - \frac{1}{5} = 7 - 0.2 = 6.8$$

$$\therefore \boxed{\bar{x} = 6.8}$$

$$\text{Mode}(M) = l + \left[\frac{f_s}{f_p + f_s}\right] \times h$$

Here ; $l = 6$; $f = 21$; $f_p = 17$; $f_s = 15$

$$\therefore M = 6 + \left[\frac{15}{17 + 15}\right] \times 2$$

$$= 6 + \frac{15}{\cancel{32}_{16}} \times 2$$

$$= 6 + \frac{15}{16}$$

$$= 6 + 0.93 = 6.93$$

∴ from standard deviation :-

$$\sigma = h\sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i d_i^2 - \left(\frac{1}{N}\sum_{i=1}^{n} f_i d_i\right)^2}$$

$$= 2\sqrt{\frac{\cancel{180}^{9}}{\cancel{80}_{4}} - \left(\frac{-\cancel{8}}{\cancel{80}_{10}}\right)^2}$$

$$= 2\sqrt{\frac{9}{4} - \frac{1}{100}}$$

$$= 2\sqrt{2.25 - 0.01}$$

$$= 2\sqrt{2.24} = 2(1.4967) = 2.98$$

$$\therefore \sigma = 2.98$$

∴ coefficient of skewness by karl pearson's

$$\therefore S_k = \frac{\text{Mean} - \text{Mode}}{\sigma} = \frac{6.8 - 6.93}{2.98}$$

$$= -\left[\frac{6.93 - 6.8}{2.98}\right] = -\left[\frac{0.13}{2.98}\right] = -[0.043]$$

$$\therefore S_k = -[0.043]$$

Inference :- karl pearson's coefficient of skewness is −0.043. which lies in ±1.

Problem :- 4

Aim :- To find Bowley's coefficient of Skewness.

Formula & procedure :-

Skewness :- It means lack of symmetry. It gives an idea about the shape of the curve of given data.

Bowley's coefficient of skewness :-

This method is based on quartiles. The formula for calculating coefficient of skewness is given by

$$\boxed{S_k = \frac{Q_3 - 2Q_2 + Q_1}{Q_3 - Q_1}}$$

$Q_1 = \frac{n}{4}$ then

$$Q_1 = l + \left[\frac{n/4 - c.f}{f}\right] \times h$$

$Q_2 = \frac{2n}{4} = \frac{n}{2}$ then

$$Q_2 = l + \left[\frac{n/2 - c.f}{f}\right] \times h$$

$Q_3 = 3\left(\frac{N}{4}\right)$ then

$$Q_3 = l + \left[\frac{3n/4 - c.f}{f}\right] \times h$$

Calculation :-

C.I	f	c.f
5-10	45	45
10-15	26	71
15-20	16	87
20-25	13	100
25-30	12	112
30-35	2	114
35-40	4	118
	$\Sigma f = 118$	

$\therefore Q_1 = \frac{N}{4} = \frac{\cancel{118}\,59}{\cancel{4}\,2} = 29.5$

$\therefore Q_1 = l + \left[\frac{n/4 - c.f}{f}\right] \times h$

Here $n/4 = 29.5^{th}$ observation belongs to 5-10

$\therefore$ c.f is 45

$\therefore Q_1 = 5 + \left[\frac{29.5 - 45}{\cancel{45}\,9}\right] \cancel{\times 5}$

$= 5 + -\left[\frac{45 - 29.5}{9}\right]$

$= 5 - \left[\frac{15.5}{9}\right] = 5 - 1.723$

$= 3.277$

$\boxed{Q_1 = 3.277}$

$\therefore Q_2 = \frac{2n}{4} = \frac{n}{2} = \frac{118^{59}}{2} = 59$

$\therefore Q_2 = l + \left[\frac{n/2 - c.f}{f}\right] \times h$

Here 59^{th} observation belongs to 10-15

$\therefore c.f = 45$; $f = 26$; $l = 10$

Here $\frac{n}{2}$ is.

$\therefore Q_2 = 10 + \frac{59 - 45}{26} \times 5$

$= 10 + \frac{14^{7}}{26_{13}} \times 5 = 10 + \frac{35}{13}$

$= 10 + 2.69$

$= 12.69$

$\boxed{Q_2 = 12.69}$

$\therefore Q_3 = \frac{3n}{4} = 3 \times 29.5 = 88.5$

Here 88.5^{th} observation belongs to 15-20

$\therefore c.f = 71$; $f = 16$; $l = 15$

$\therefore Q_3 = l + \left[\frac{3n/4 - c.f}{f}\right] \times h$

$= 15 + \left[\frac{88.5 - 71}{16}\right] \times 5$

$= 15 + 5.46$

$= 20.46$

$\therefore \boxed{Q_3 = 20.46}$

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∴ Bowley's coefficient of skewness is

$$S_k = \frac{Q_3 - 2Q_2 - Q_1}{Q_3 - Q_1}$$

$$= \frac{20.46 - 2[12.69] - 3.277}{20.46 - 3.277}$$

$$= \frac{20.46 - 25.38 - 3.277}{20.46 - 3.277}$$

$$= \frac{-8.19}{17.18} = -0.4767$$

$$\boxed{\therefore S_k = -0.4767}$$

<u>Inference</u> :- Bowley's coefficient of skewness is -0.4767 which lies in the ±1

Practical - 6

Probability

Problem :- 1

To find if two dice are thrown, what is the probability that the sum is either 10 or 11.

Problem :- 2

To find if a box contains 20 tickets Numbered from 1 to 20. A ticket is drawn at random Find the probability that it is divisible by 3 or 4.

Problem :- 3

To find the if A bag contains 4 red ball and 5 black. If two balls are drawn in succession, then find the probability that they are red balls if the drawn ball at find draw was i) replaced ii) Not replaced

Problem :- 4

The contents of urns I, II & III as follow

urn I : 1 white 2 black and 3 red balls

urn II :- 2 white 1 black and 1 red ball

urn III :- 4 white 5 black and 3 red ball

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one urn is chosen at random and two balls are drawn, they happen to be white and red. Find the probability that they come from urns I, II or III

Problem :- 5

In a bolt factory machines A, B & C manufactures 20%, 30% and 50% respectively of the total of their total output, 6%, 3% and 2% are defective. A bolt is drawn at random and found to be defective. Find the probabilities that it is found to be manufactured by Machines A, B & C.

Problem :- 1

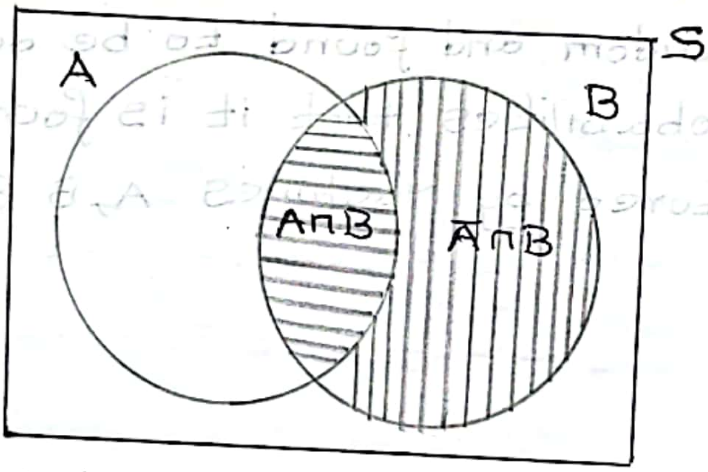
Aim :- To find if two dice are thrown, what is the probability that the sum is either 10 or 11

Formula & Procedure :-

Additional theorem :-

If A & B are any two events then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



Calculation :-

If two dice are thrown, then the No. of outcomes $n(S) = 36$

Let Event A be getting the sum is 10

Favourable outcomes are $\{(4,6),(5,5),(6,4)\}$;

$\therefore n(A) = 3$

Let B be getting the sum is 11

$\therefore$ Favourable outcomes are $\{(5,6),(6,5)\}$;

$\therefore n(B) = 2$

$\therefore A \cap B = \phi$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{36}$; $P(B) = \frac{n(B)}{n(S)} = \frac{2}{36}$

$\therefore$ P(getting the sum is either 10 or 11) = $P(A \cup B)$

By using Additional theorem

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{3}{36} + \frac{2}{36} - 0 = \frac{5}{36}$$

Inference :- We conclude the probability that sum is either 10 or 11 is $\frac{5}{36}$.

Problem :- 2 :-

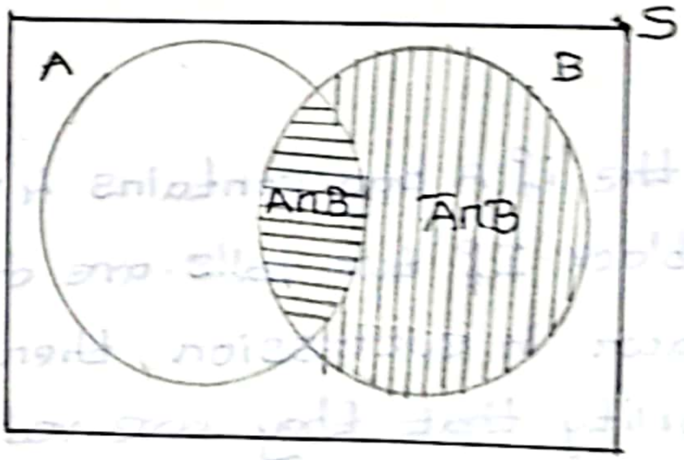
Aim :- To find if a box contains 20 tickets Numbered from 1 to 20. A ticket is drawn at random. Find the probability that it is divisible by 3 or 4.

Formula & Procedure :-

Additional theorem :-

If A & B are any two events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Calculation :- Tota No. of outcomes $n(S) = {}^{20}C_1 = 20$.

Let A be the No. is divisible by 3

$\therefore A = \{3, 6, 9, 12, 15, 18\}$ $\therefore n(A) = {}^6C_1 = 6$

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{20}$$

Let B be the No. is divisible by 4

$\therefore B = \{4, 8, 12, 16, 20\}$ $\therefore n(B) = {}^5C_1 = 5$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{5}{20}$$

$A \cap B$ is the No. is divisible by both 3 & 4

$\therefore A \cap B = \{12\}$ $\therefore n(A \cap B) = 1$

$$\therefore P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{20}$$

By using Additional theorem of probability

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{20} + \frac{5}{20} - \frac{1}{20}$$

$$= \frac{10}{20} = \frac{1}{2}$$

Inference :- We conclude the probability that it is divisible by 3 or 4. is $\frac{1}{2}$

Problem :- 3 :-

Aim :- To find the if A bag contains 4 red ball and 5 black. If two balls are drawn are drawn in succession, then find the probability that they are red balls If the drawn ball at find draw was i) replaced ii) Not replaced.

Formula & Procedure :-

Multiplication Theorem :-

For two events A & B

$$P(A \cap B) = P(A) \cdot P(B/A) \;;\; P(A) > 0$$
$$= P(B) \cdot P(A/B) \;;\; P(B) > 0$$

where $P(B/A)$ is conditional probability of B given A, $P(A/B)$ is conditional probability of A given B.

Calculation :-

i) The drawn ball is replaced before the second draw

Let A be drawing red ball at first draw

B be drawing red ball at second draw

For the event, A

$n = {}^9C_1 = 9$

$m = {}^4C_1 = 4$

$\therefore P(A) = 4/9$

For the event, B

$n = {}^9C_1 = 9$ [$\because$ since ball was replaced]

$m = {}^4C_1 = 4$

$\therefore P(B) = 4/9$

$\therefore$ P (drawing balls are red balls in two draws)

$= P(A \cup B) = P(A) \cdot P(B)$ [$\because$ A, B are independent Events]

$= \frac{4}{9} \cdot \frac{4}{9} = \frac{16}{81}$

ii) The drawn ball is not replaced before the 2nd draw. Let A be drawing a red ball at first draw and Let B/A be drawing a red ball at 2nd draw after the red ball drawn at first draw.

For the event A,

$n = {}^9C_1 = 9$

$m = {}^4C_1 = 4$

$\therefore P(A) = 4/9$

For the event B/A,

$n = {}^8C_1 = 8$ [$\because$ since ball Not replaced]

$m = {}^3C_1 = 3$

$\therefore P(B/A) = \frac{3}{8}$

By using Multiplication Theorem,

P (drawing two red balls in successive draws)

$= P(A \cap B) = P(A) \cdot P(B/A)$ [$\because$ A,B are dependent event]

$= \frac{\cancel{4}}{\cancel{9}_3} \cdot \frac{\cancel{3}}{\cancel{8}_2}$

$= \frac{1}{6}$

Inference :- We conclude that the probability that they are red balls if the drawn ball at draw was replaced and Not-replaced is $\frac{3}{8}$ & $\frac{1}{6}$

Problem :- 4 :-

Aim :- The contents of urns I, II and III as follow.

urn I : 1 white, 2 black and 3 red balls

urn II : 2 white, 1 black and 1 red balls

urn III : 4 white, 5 black and 3 red balls

one urn is chosen at random and two balls are drawn, they happen to be white and red. Find the probability that they come from urns I, II or III

Formula & Procedure :-

Baye's Theorem :-

Suppose $E_1, E_2, \ldots, E_n$ are n Mutually Exclusive and Exhaustive events of a random Experiment with $P(E_i) \neq 0$ then

$$P(E_k/A) = \frac{P(E_k) \cdot P(A/E_k)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}$$

Calculation :- Let E_1 : Selecting urn I

E_2 : selecting urn II

E_3 : Selecting urn III

A : drawing two balls which are white & red

$P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{1}{3}$, $P(E_3) = \frac{1}{3}$

$P(A/E_1) = \frac{1c_1 \cdot 3c_1}{6c_2} = \frac{1}{5}$

$$P(A/E_2) = \frac{2c_1 \cdot 1c_1}{4c_2} = \frac{1}{3}$$

$$P(A/E_3) = \frac{4c_1 \cdot 3c_1}{12c_2} = \frac{2}{11}$$

$$P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)$$

$$= P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)$$

$$= \frac{1}{3} \cdot \frac{1}{5} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{2}{11}$$

$$= 118/495$$

By using Baye's Theorem,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(A)}$$

$$= \frac{\frac{1}{3} \cdot \frac{1}{5}}{118/495} = \frac{33}{118}$$

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(A)}$$

$$= \frac{1/3 \cdot 1/3}{118/495} = \frac{55}{118}$$

$$P(E_3/A) = \frac{P(E_3) \cdot P(A/E_3)}{P(A)}$$

$$= \frac{1/3 \cdot 2/11}{118/495} = \frac{30}{118}$$

Inference :- We conclude that they probability that they come from urns I, II or III is $33/118$, $55/118$, $30/118$

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Problem :- 5

Aim :- In a bolt factory machines A, B & C Manufactures 20%, 30% & 50% respectively of the total of their total output, 6%, 3% & 2% are defective. A bolt is drawn at random and found to be defective. Find the probabilities that it is found to be manufactured by Machines A, B & C.

Formula & Procedure :-

Baye's Theorem :-

Suppose $E_1, E_2, \ldots, E_n$ are n Mutually Exclusive and Exhaustive events of a random Experiment with $P(E_i) \neq 0$ then

$$P(E_K/A) = \frac{P(E_K) \cdot P(A/E_K)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}$$

Calculation :- Let E_1 : a bolt is Manufactured by Mach A

E_2 : a bolt is Manufactured by Machine B,

E_3 : a bolt is Manufactured by Machine C,

A : a defective bolt is found

Given that $P(E_1) = 0.2$; $P(E_2) = 0.3$; $P(E_3) = 0.5$

$P(A/E_1) = 0.06$; $P(A/E_2) = 0.03$; $P(A/E_3) = 0.02$

$$\therefore P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)$$

$$= P(E_1) \cdot P(A/E_1) + P(E_2) P(A/E_2) + P(E_3) \cdot P(A/E_3)$$

$$= (0.2)(0.06) + (0.3)(0.03) + (0.5)(0.02)$$

$$= 0.031$$

By using Baye's theorem,

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{P(A)}$$

$$= \frac{(0.2)(0.06)}{0.031} = 0.3871$$

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(A)}$$

$$= \frac{(0.3)(0.03)}{0.031} = 0.2903$$

$$P(E_3/A) = \frac{P(E_3) \cdot P(A/E_3)}{P(A)}$$

$$= \frac{(0.5)(0.02)}{0.031} = 0.3226.$$

Inference :- We conclude the probabilities manufactured by machines A, B & C